Quo vadis thermodynamics and the city: a critical review of applications of thermodynamic methods to urban systems

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Abstract
Cities are complex dynamic entities. From a thermodynamic perspective, they represent open systems, constantly importing and exporting energy and matter across their boundaries; the output invariably being of less ordered due to irreversible internal processes [dissipation]. In analogy to natural ecosystems, the uptake, transport and storage of substances in urban sites, as well as their transformation, can be considered as equivalent to urban metabolism.

Apparently inspired by progress that has been made in the study of natural ecosystems, a range of tools based on thermodynamics have recently been applied to better understand this urban metabolism and how it might be optimised. In this paper we review progress that has been made in both domains (that is, natural and anthropogenic ecosystems). In particular we consider applications of entropy, energy, exergy and (the quasi-thermodynamic) emergy. In this we attempt to qualitatively define a seemingly general dichotomy between the explanatory power of a given thermodynamic concept and its calculability.

We conclude that, whilst thermodynamic considerations indisputably provide a general framework for understanding cities and their evolution, further research is needed for them to become operational representations of urban metabolism and indicators of sustainability. We suggest possible pathways for achieving this.

Keywords: city, thermodynamics, open system, entropy, sustainable development, evaluation.
1 Introduction

Unlike natural ecosystems, in which myriad organisms coexist in a process of synergetic exchange so that resource flows are highly circular; in anthropogenic systems (cities) they are essentially linear. We (humans) import resources (food, fuel, raw materials), combust and / or process them and export the waste products with almost complete disregard for the environmental consequences. Since half of the global population is currently urbanised the environmental consequences of this resource inefficiency, such as greenhouse gas emissions and raw material depletion, are alarming. Indeed, with the urban population forecast to increase to three fifths during the next twenty five years, the future looks somewhat bleak. It is clearly important than that we better understand this urban metabolism, its consequences and ways in which it can be optimised to minimise these consequences.

In reflecting on the mysteries of life Schrödinger [1] noticed that, rather than dissipate order (or increase entropy) and so decay to thermodynamic equilibrium (death), living organisms actually manage to increase or maintain their degree of organisation or order by consuming negentropy. Put more formally, we may say that an organism may be regarded as an open system, in the sense that energy and matter are exchanged across its boundaries. This openness and the presence of flows further imply that the system is also a non-equilibrium one. In other words, whereas the entropy of isolated physicochemical systems tends to evolve irreversibly to equilibrium (\(dS > 0\)), open systems may be maintained in non-equilibrium states. The entropy change \(dS\) in such a system is then \(dS = dS_i + dS_e\), where \(dS_i\) corresponds to entropy production due to irreversible internal processes (e.g. human cellular dynamics\(^1\)) and \(dS_e\) to entropy exchange across the system boundaries (we humans consume solid and liquid food, breathe in Oxygen and expel Carbon Dioxide and other waste products)\(^2\).

Prigogine [3] later formalised these observations, which apply to a great many systems, creating a new branch of scientific study – modern or far from equilibrium thermodynamics. In 1977 Nicolis and Prigogine [4] suggested that cities may be considered as one such system, noting that a city “can survive as long as it is the inflow of food, fuel and other commodities and sends out products and waste”. This then is an example of what later became known as a dissipative structure; referring to the “creation and maintenance of organised non-equilibrium states due to dissipative processes” [5].

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\(^1\) For instance the rapid increase in self organisation during the early stages of growth from foetus through new-born child to young infant is quite remarkable.

\(^2\) Indeed Schneider and Kay [2] suggest that the more energy that is ‘pumped’ through a system, the greater the degree of organisation that can emerge to dissipate this energy. They go on to suggest that the two processes are inseparable, so that if a structure is not ‘progressive’ enough it will be replaced by a better adapted structure that will use the available energy more effectively.
1.1 Thermodynamics and the city

Since Prigogine’s initial observation significant progress has been made in the application of thermodynamics to the study of urban development.

One of Prigogine’s collaborators, Allen [6, 7] pioneered the application of non-linear dynamics with bifurcations to study the spatial dynamics of urban land use. The interactions between different urban actors and their basis for decision making led to the creation of a functional structure for the city which supported investigations into the dynamics of the evolution of urban land use. Allen’s work may be regarded as the forerunner to subsequent (microscopic) studies of urban dynamics, based for example on cellular automata and agent based modelling. In this respect Batty [8] has experimented with different neighbourhood rules for mimicking urban growth using cellular automata and Portugali [9] and Benenson and Torrens [10] have used agent based models to examine urban re-location and land use change.

More recently some workers have also investigated the application of thermodynamics to sustainable urban development.

Machettini et al [11] for example discuss two extreme scenarios of an urban system: autarchy and globalisation; arguing that these regimes are not thermodynamically viable. The first approaches what is known as an isolated system, in which interactions and exchanges with the systems environment are inhibited. This is known to lead to a fading out of living processes. The second represents an extreme case of an open system that relies on over-exploitation of planetary resources. This regime is deemed to be inconsistent with physical laws, since it leads to exhaustion of natural capacity: it is self-destructive. Although non-viable these two extremes may usefully be applied to define the bounds of a liveability space. What remains then is to define a quantitative representation of these extremes as well as of the state of some real urban system that enables it to be placed somewhere between the extremes.

This leads us to the real objective of this paper: to investigate the extent to which the scientific framework of thermodynamics can now be applied to represent urban metabolism in an operational way as well as the extent to which thermodynamics can be used to help us quantitatively determine urban sustainability (sustainability indicators). For this we refer to the relatively rich literature on ecosystem theory and to the relatively sparse literature on urban metabolism. As Wilson [12] puts it “learning about ecosystems and the (fundamentally thermodynamic) laws of their evolution can help us to

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3 Modelling techniques that have been applied to numerous complex systems, in fields as diverse as cellular dynamics, fluid dynamics and ecosystem dynamics.

4 To clarify: derived from the Greek for self-governance autarchic may also be taken to mean independent. We (humans) may be autonomous but we cannot be independent. To live sustainably ourselves we need relationships with other species; we also need to import solar energy.

5 Although some may argue that human society is intrinsically self-destructive.
understand cities in these terms and draw analogies in terms of sustainability between urban and natural habitats”.

1.2 Quantifying sustainability: general criteria on indicators

Before embarking upon on a critical review of the developments that have been made in these separate but related domains, let us define some general criteria with which we might judge the utility of a particular (quantitative as distinct from qualitative) indicator of sustainability (the subject which has thus far attracted the majority of theoretical progress). We suggest the following:

**Measurability / calculability**: An indicator should by definition be quantitative. One must therefore be able to empirically measure or to calculate this quantity.

**Diagnostic ability**: It is important to be able to isolate causal mechanisms for a particular indicator value (or a change in this value). This is particularly relevant for indicators that are based on the aggregation of low-level data (it should be possible to trace back to the appropriate level and variable responsible).

**Succinctness** (Occam’s razor): The indicator should make as few assumptions about the phenomenon in question as possible and elements which have no observable impact on results should be eliminated.

An indicator should also be constructive: “it should tell us what to do, not only what not to do. Sustainability policy needs proactive assessment indicators to orient its activities” (Newman [13]). We suggest that a good indicator should be constructive but that it must satisfy the above three criteria.

2 Applications of thermodynamics to ecosystems

In his review of the state-of-the art in ecosystem theory Müller [14] classifies ecological thermodynamics as “correct in a very general sense”. He discusses the conceptual framework provided by thermodynamics (the 1st and 2nd laws, the notion of open systems and the phenomenon of self-organisation). He also briefly reviews the main extrema ecological principles that are based on thermodynamics but were devised in the field of ecology: the non-equilibrium principle (Schneider and Kay [2]), the ecological principle of thermodynamics (Jørgensen [15]), the principle of emergy optimisation (the maximum empower principle) (Odum [16]), the principles of optimisation of total entropy production and of specific entropy. Müller goes on to state that all these principles (or hypotheses of ecosystem development) lead to similar conclusions:

“the more order an ecosystem obtains the lower is its entropy state, the greater is its distance from thermodynamic equilibrium, the higher is the information content, structural complexity, exergy flow and the exergy state and the higher are the total costs for the conservation of complexity and for entropy production”.

He sees thermodynamics and principles based on thermodynamics as “a starting point for a further integration of ecosystem theories” in complement with
theories of self-organisation. Thus far, particular progress has been made in respect of exergy and entropy.

2.1 Exergy

For Exergy (a measure of the ability of a system to perform work) Jørgensen [15, 17, 18] proposed the Exergy Maximum Principle, which states that “an ecosystem strives towards the highest possible exergy at the prevailing conditions”. In other words the exergy content of an ecosystem increases as it evolves. The principle further suggests that if a system receives an input of exergy, it will utilise this exergy to perform work. The work performed: i) degrades the exergy, dissipating the residue as entropy to the system’s surroundings, (ii) moves the system further from thermodynamic equilibrium, reflected in the growth of gradients, and (iii) increases the accumulated energy-mass of the system, representing additional stored exergy (Patten and Jørgensen, [18]).

Furthermore, if more than one pathway is offered to depart from equilibrium, the one yielding the highest gradient, the highest throughput (maximum power) and the most storage of exergy under the prevailing conditions, to give the most ordered structure furthest from equilibrium, will tend to be selected.

Exergy thus provides a measure both of the fitness of an ecosystem (Exergy Maximum) and its derivation has helped to deepen our understanding regarding the principles by which natural ecosystems evolve. What is not clear however, is to what extent this framework may be applied to anthropogenic systems in which external influences (e.g. climate) may be compensated for artificially by importing more resource (e.g. energy). Structural adaptations are only required to take place in such circumstances if imports are limited (e.g. we must use a reduced availability of energy more efficiently). The Exergy of an urban system may be calculable and it may be succinct, but is it diagnostically useful?

2.2 Entropy

Svirezhev [19] introduced an elegant concept called “the entropy pump” in application to agricultural ecosystems. In this, the state of a human-modified system is contrasted, by the addition of “artificial” energy, to that of a natural ecosystem (of corresponding succession type). The total entropy balance of an agro-system is written as:

\[ \frac{dS}{dt} = \frac{1}{T} \left[ GPP_t + W - GPP_0 \right] = \sigma \]

where \( GPP_0 \) is the gross primary production\(^6\) (J) of the natural ecosystem and \( W \) is the artificial energy (J), including energy and chemical inflows (or anthropogenic pressure), added to the human-modified system of \( GPP_t \) and \( T \) is a mean annual temperature (K). Now if \( \sigma > 0 \) the system overproduces entropy so that the

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\(^6\) Primary production is defined as the synthesis and storage by photosynthetic organisms of organic molecules during growth and reproduction and \( GPP \) refers to the corresponding Carbon that is fixed.
environment is degraded\textsuperscript{7}. In principle then, we are able to find the limiting energy load on an agro-system to achieve sustainable yield.

This elegant model allows us to calculate the entropy of a non-equilibrium system under anthropogenic pressure without having to directly calculate the entropy due to individual physical processes. It also provides a measure of the degree of the system’s environmental degradation. Svirezhev demonstrates how this model may be implemented in an agricultural context. The challenge in our context is to characterise what might constitute $GPP_0$ for an urban settlement. One option might be the hypothetical autarchic system of Machettini at al \cite{11} or some thermodynamically viable \textit{utopian} closed urban system. The entropy pump hypothesis thus seems to be diagnostically useful, it is succinct and it may well be calculable.

A very comprehensive (and, to the knowledge of the authors, the single pure) application of non-equilibrium thermodynamics in the non-linear regime to natural (aquatic) ecosystems was developed in a series of works by Mauersberger \cite{20, 21}. He proposed a thermodynamic theory of aquatic ecosystems as a complex system of non-linear differential equations describing the evolution of an ecosystem and its population dynamics. In this he proposes a complex thermodynamically-based framework comprising a set of physical-chemical energy and mass balance equations with rates of biological processes as functions of state variables.

With this work ecology, building on the principles of far from equilibrium thermodynamics, has a sound theoretical basis. However, the practical application and calculation of entropy production, based on this fundamental approach, remains highly difficult. It requires calculation of all thermodynamic flows and forces with subsequent integration over the system volume: a problem which involves too many variables, parameters and uncertainties in natural as well as human ecosystems. This limitation thus renders this fundamental approach inappropriate according to our criteria.

\section{Applications to urban studies}

In the last decade a range of attempts have been made to apply thermodynamically-inspired (as distinct from truly thermodynamic) principles to analyse the sustainability of urban systems. Of these, Emergy and Statistical Entropy appear to have been most commonly employed.

\subsection{Emergy}

Emergy is available energy of one kind that has been used up to make a product or service (Odum, \cite{16}), expressed in terms of solar energy units (solar energy joules; seJ), using transformities (seJ/J) to convert from conventional energy

\textsuperscript{7} Note that in natural ecosystems (dynamic in the short term, but in long-term steady state) this over-production is zero as this entropy is sucked-out by the solar entropy pump (equivalent to Schrödinger’s negentropy).
units (J). These transformities are the key parameters in emergy calculations. When performing emergy analyses, the various stocks and flows are often represented using an Energy Systems Diagram, which helps to structure a potentially complex web of processes. Developed from analyses of natural ecosystems, these diagrams may also qualitatively represent steps in the processes of energy degradation which accompany its transformation – as the primary energy flows from source to sink, so depicting an energy hierarchy [22].

Huang et al have repeatedly applied the emergy method to the analysis of urban systems [23, 24, 25]. As part of their more recent work [25] they use the product of urban production $^9$ (seJ/kg)) and a constant calibration coefficient to quantify urban waste generation. This they further define as entropy (seJ/day), which is also interpreted as the “emergy value of waste flows”. Observing its increase over forty years the authors anecdotally conclude that “urban-industrial civilisation is vastly accelerating the process of entropy”. Whilst the simplicity of this calculation may be seductive, the proposed entropy has no physical basis. It also seems to be logically flawed. Consider two cities: city one is larger than city two, likewise the population and the environmental stocks. But city two is energy profligate, such that total waste production (or entropy) is similar. However, if more energy is expended to maintain similar thermal gradients (particularly if normalised in terms of built area) then we would naturally conclude that city two produces higher entropy than city one.

Considering once again our three criteria of a good indicator, then in principle emergy is calculable, but the centrepiece transformity is shrouded in uncertainty (and sometimes lacks physical meaning). If we take calculability to imply some degree of reasonable precision then this criterion may not be satisfied. Furthermore, the aggregate results of emergy may submerge the factors which are responsible for a given result – interpretation may be ambiguous. Finally, this method seems to fail the test of succinctness. It is a combination of an accounting method (no different in principle to conventional material flow analysis, though in some ways less rigorous: see e.g. [26]) and a means of conversion of the resultant energy and mass flows; which is not always required and not necessarily in emergy units. But perhaps most important is the lack of physical meaning: implicit in succinctness is that the theory should encapsulate the physical essence of the phenomenon under investigation.

3.2 Statistical Entropy

Our final port of call concerns the use of statistical or information (as opposed to thermodynamic) entropy in the urban context. These calculations tend to be based on Shannon’s equation for the information entropy of a given sequence of binary digits: $H(X) = -\sum_{i=1}^{n} p(x_i) \log_2 p(x_i)$; where $H(X)$ is the information

$^9$ Itself the product of urban area, built floor area, environmental stocks, energy use and population.
Entropy of discrete variable $X$ and $p(x_i) = P(X = x_i)$ is the probability mass function of $X$ [27]. Besides the similarity in the expressions for information entropy and the statistical Boltzmann entropy in physics, there is in principle a link between this information theory and thermodynamics, if we define entropy as thermodynamic information. That is “all structural changes, fluxes of energy and all matter transfers are coupled with information flows” [14]. Unfortunately it is not in this respect that information entropy has thus far been applied.

In common with thermodynamics, information theory has been applied in a more systematic way in ecosystems science; for example to indicate population / species (or bio-) diversity (see e.g. indicators of Shannon-Wiener and Margalef in [14]) and as a measure of the diversity of landscape patterns (see e.g. Antrop [28]). This latter is based on analysis of the spatial patterns of maps, the abundance of elements and their diversity, to characterise landscape changes according to the corresponding entropy. High entropy was taken to symbolise disorder (high biodiversity) and low values indicated high order (low biodiversity) of the landscape patterns. This method is easily calculable. It expresses overall global diversity whilst preserving lower level data, but it does not indicate the driving forces for change (system behaviour is not represented) in landscape order, so that it is not diagnostically useful. Furthermore, this indicator expresses only the quantity not the quality of species diversity. Finally, and despite the above drawbacks, it is not obvious how this approach can be meaningfully applied to the urban context in which we’re interested not in biodiversity but in environmental (thermodynamic) degradation. Nevertheless some attempts have been made:

Referring to open systems, dissipative structures and complex systems theory, Zhang et al [29, 30] propose a Shannon-like indicator of urban sustainability (the degree of urban order or health), which they suggest is rooted in thermodynamics: $dS = - \frac{1}{\ln(j = m)} \sum_{i=1}^{n} \frac{q_{ij} \ln q_{ij}}{q_{ij}}$; where $q_{ij}$ is one of 42 normalised indicators used to describe urban energy and matter flows; each of which are similarly weighted. The rationale for their inclusion is also unclear. Again this is calculable, but it is neither succinct nor diagnostically useful.

A similar approach has been employed by Rechberger [31] to indicate the entropy (or contaminant dilution) within an industrial (materials processing) context; to which similar criticisms may be levelled.

4 Discussion and Conclusions

On reflecting on the various thermodynamically inspired means of evaluation discussed in this paper, we suggest that there is something of a dichotomy between theoretical rigour and practical feasibility. More specifically we suggest that the calculability of (quasi) thermodynamic expressions is inversely related to their aggregation level, succinctness and diagnostic power. In other words, quantities that are easily calculated do not require strong hypotheses. In the
inverse case, quantities requiring strong hypotheses also tend to be difficult to measure; but they have considerable greater explanatory power (Figure 1).

Efforts to apply thermodynamic (or thermodynamically inspired) concepts to the study of urban metabolism have to date been based on somewhat questionable theoretical foundations; whereas many of the studies from the field of ecology are rigorous and offer good potential for application to urban systems. This is particularly the case with respect to Svirezhev’s entropy pump hypothesis and also (though perhaps to a lesser extent) to Jørgensen’s exergy analysis.

These principles should help us to understand and evaluate urban metabolism, though not necessarily to encapsulate it at a functional level through mathematical models. Perhaps the thermodynamically influenced models of the dynamics of urban growth (s2.1) will provide food for thought here?

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5 References


